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Finite electromagnetic mass difference of pions and tensor gravity

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Abstract. It is shown how the tensor gravity makes the electromagnetic mass splitting of pions finite without using any lagrangian model for the strong interaction. The mass splitting is sensitive to the spectral-function sum rules. A finite mass splitting does not lead to a unique modification of the second Weinberg sum rule. For the electromagnetic mass difference of pions good values are obtained by suitably chosen input data.

1. Introduction

After the first current algebra calculation of the electromagnetic mass splitting of pions (Das *et al* 1967) the presence of a logarithmic divergence has been shown for on-shell external pions (Gerstein *et al* 1967) and also in the soft-pion limit with massive-pion contributions to the spectral-function sum rules (Cook *et al* 1968, Albright *et al* 1967). A finite and reasonable result could be found by modifying the second Weinberg sum rule (Cook *et al* 1968). It is, however, not at all excluded that instead of the generalization of Cook *et al* a slightly different sum rule is valid leading essentially to the same mass of A_1 and the logarithmic divergences do not cancel. This shows that a regularizing mechanism is necessary to get finite mass shifts even if complete cancellations occur in the last step.

In a recent paper, starting from the proposal that gravity provides a natural regularization for the self-mass of the electron (Salam and Strathdee 1970a, Isham *et al* 1971), we have found an acceptable value for the mass shift of pions in gravity-modified quantum electrodynamics (Farkas and Pócsik 1972) and similar conclusions have also been drawn in lagrangian models of the strong interaction (Duff *et al* 1971, Huskins 1972). In the present paper we include the strong interaction in a model-independent way and express the mass shift of pions in terms of spectral functions and the gravitational constant. This is carried out in the soft-pion limit in § 2. In § 3 we show and examine the dependence of the mass shift on the spectral-function sum rule. In one particle approximation we find two kinds of solutions. One of them presents the result of Cook *et al*, here, the gravitational constant κ gives only negligible contributions. The other solution corresponds to second Weinberg sum rules modified slightly differently from that of Cook *et al* and the gravitational terms give large contributions. Section 4 contains a discussion of the results.

2. Derivation of the mass shift of pions

We introduce gravity only into the electromagnetic vertices

$$\mathscr{L}_\gamma = -\sqrt{-g}A_\mu(j^\mu + e^2\phi^+\phi A^\mu), \qquad g = \det g_{\alpha\beta}. \tag{1}$$

The violation of the electromagnetic gauge invariance caused by (1) is only of the order κ (see also Farkas and Pócsik 1972), similarly the gravitational gauge dependence is not strong (see below). From (1) the electromagnetic self-energy of the pion (Umezawa 1958, Das *et al* 1967) to e^2 order follows as

$$\Delta E_\pi = (2\pi)^3 \operatorname{Re}\frac{1}{2\mathrm{i}}\int \mathrm{d}^4x D_{\mu\rho}(x)S^{\mu\nu\rho\sigma}(x)\{\langle\pi|T(j_\nu(x)j_\sigma(0))|\pi\rangle - \langle 0|T(j_\nu(x)j_\sigma(0))|0\rangle\} \tag{2}$$

where $D_{\mu\rho}(x)$ is the free-photon propagator in Fried–Yennie gauge,

$$S^{\mu\nu\rho\sigma}(x) = \langle 0|T(g^{\mu\nu}(x)\sqrt{-g(x)}g^{\rho\sigma}(0)\sqrt{-g(0)})|0\rangle \tag{3}$$

means the graviton superpropagator and the rest contains the strong interaction. The second-order electromagnetic mass splitting of π^+ and π^0 is determined by the isovector part

$$\delta\mu^2 = 2m_\pi(2\pi)^3 e^2 \operatorname{Re}\frac{1}{2\mathrm{i}}\int \mathrm{d}^4xD_{\mu\rho}(x)S^{\mu\nu\rho\sigma}(x)\{\langle\pi^+|T(V_\nu^3(x)V_\sigma^3(0))|\pi^+\rangle - (\pi^+\leftrightarrow\pi^0)\}, \tag{4}$$

V_γ^{i} is the isovector current.

Now, proceeding as Das *et al* we reduce both of the pions, using PCAC and partial integrations together with current commutators, we get in the soft-pion limit

$$\delta\mu = \frac{e^2}{2(2\pi)^8 m_\pi F_\pi^2}\operatorname{Re}\int \mathrm{d}^4q\, \mathrm{d}^4p \frac{1}{q^2+\mathrm{i}\epsilon}\left(\eta_{\mu\rho} - 4\frac{q_\mu q_\rho}{q^2}\right)\tilde{S}^{\mu\nu\rho\sigma}(p+q)(\Delta_{\nu\sigma}^{\mathrm{V}}(p) - \Delta_{\nu\sigma}^{\mathrm{A}}(p)). \tag{5}$$

$\tilde{S}^{\mu\nu\rho\sigma}(p)$ denotes the Fourier transform of the graviton superpropagator (3),

$$(\eta_{\mu\rho}) = + - - -$$

and $\Delta_{\nu\sigma}^{\mathrm{V}} - \Delta_{\nu\sigma}^{\mathrm{A}}$ is well known (Cook *et al* 1968)

$$\begin{aligned}\Delta_{\nu\sigma}^{\mathrm{V}}(p) - \Delta_{\nu\sigma}^{\mathrm{A}}(p) = -\mathrm{i}\Bigg[\int_0^\infty \frac{\mathrm{d}m^2}{-p^2+m^2-\mathrm{i}\epsilon}\Bigg\{&\left(\eta_{\nu\sigma} - \frac{p_\nu p_\sigma}{m^2}\right)\\ &\times(\rho_{\mathrm{V}}(m^2) - \rho_{\mathrm{A}}(m^2)) + \frac{p_\nu p_\sigma}{m^2}\Delta\rho_{\mathrm{A}}(m^2)\Bigg\} + F_\pi^2\frac{p_\nu p_\sigma}{-p^2+m_\pi^2-\mathrm{i}\epsilon}\Bigg],\end{aligned} \tag{6}$$

with $\rho_{\mathrm{V}}(\rho_{\mathrm{A}})$ the transversal vector (axial vector) Lehmann weight and $\Delta\rho_{\mathrm{A}}$ the longitudinal axial vector weight function without the pion contribution. Equation (5) reproduces the result of Das *et al* for $g_{\mu\nu} = \eta_{\mu\nu}$ and it can be considered as the simplest extension including gravity. Instead of the free photon propagator, equation (5) is determined by the dressed propagator

$$d^{\nu\sigma}(p) = \frac{1}{(2\pi)^4}\int \frac{\mathrm{d}^4q}{q^2+\mathrm{i}\epsilon}\left(\eta_{\mu\rho} - 4\frac{q_\mu q_\rho}{q^2}\right)\tilde{S}^{\mu\nu\rho\sigma}(p+q). \tag{7}$$

The graviton superpropagator must be treated by the methods of nonpolynomial field theories. For $g^{\mu\nu}(x)$ we choose the localizable exponential parametrization (Lehmann and Pohlmeyer 1971)

$$g^{\mu\nu}(x) = (\exp \kappa h(x))^{\mu\nu}, \qquad \kappa^{-1} = 2\times 10^{18}\,\mathrm{GeV}, \tag{8}$$

where $h_{\mu\nu}(x)$ is the free graviton field and

$$\langle 0|T(h_{\mu\nu}(x)h_{\rho\sigma}(0))|0\rangle = \tfrac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-2c\eta_{\mu\nu}\eta_{\rho\sigma})D_{\mathrm{F}}(x) \tag{9}$$

in the gauge fixed by the number c. An expression for (3) with (8) has been derived (Ashmore and Delbourgo 1971) in terms of two given entire functions G and H

$$S^{\mu\nu\rho\sigma}(x) = \eta^{\mu\nu}\eta^{\rho\sigma}G(\kappa^2 D_{\mathrm{F}})+(\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho})H(\kappa^2 D_{\mathrm{F}}) \tag{10}$$

and it follows that

$$\begin{aligned} &G(0) = 1, \qquad H(0) = 0 \\ &G(u) = H(u) = u^{3/2}\exp\{2u(1-c)\}, \qquad u\to\infty. \end{aligned} \tag{11}$$

Introduce the usual representation (Salam and Strathdee 1970b) for G and H with contour integrals and take their Fourier transforms, then we have the minimal representation

$$\begin{aligned} &\tilde{S}^{\mu\nu\rho\sigma}(p) = \eta^{\mu\nu}\eta^{\rho\sigma}\tilde{G}(p)+(\eta^{\mu\rho}\eta^{\nu\sigma}\eta^{\mu\sigma}\eta^{\nu\rho})\tilde{H}(p) \\ &\begin{pmatrix}\tilde{G}(p)\\ \tilde{H}(p)\end{pmatrix} = 8\pi^3\int_{a-\mathrm{i}\infty}^{a+\mathrm{i}\infty}\frac{\mathrm{d}z}{\sin\pi z\tan\pi z}\,\frac{\Gamma(z+1)}{\Gamma(z)\Gamma(z-1)}(-p^2)^{z-2}\left(\frac{\kappa}{4\pi}\right)^{2z}\begin{pmatrix}g(z)\\ h(z)\end{pmatrix} \end{aligned} \tag{12}$$

where $-1 < a < 0$, $g(0) = 1$, $h(0) = 0$ independently of c. It is easy to verify that the choice $g(z) \equiv 1$, $h(z) = 0$ gives exactly the superpropagator of the scalar gravity in the nonlocalizable parametrization $g^{\mu\nu} = \eta^{\mu\nu}(1+\kappa h)$.

Substituting (12) into (7) we get for the effective photon propagator the contour integral

$$d^{\nu\sigma}(p) = \frac{\pi}{2\mathrm{i}}\int_{a-\mathrm{i}\infty}^{a+\mathrm{i}\infty}\frac{\mathrm{d}z}{\sin\pi z\tan\pi z}\left(\frac{\kappa}{4\pi}\right)^{2z}(-p^2)^{z-1}\,\frac{g(z)+h(z)}{\Gamma(z)}\left(\eta^{\nu\sigma}-4\frac{p^\nu p^\sigma}{p^2}\right)\frac{z-1}{z+1}. \tag{13}$$

Put (6) and (13) into (5) and use the first Weinberg sum rule

$$\int_0^\infty \frac{\mathrm{d}m^2}{m^2}(\rho_{\mathrm{V}}(m^2)-\rho_{\mathrm{A}}(m^2)-\Delta\rho_{\mathrm{A}}(m^2)) = F_\pi^2 \tag{14}$$

then $\delta\mu$ takes the form

$$\begin{aligned} \delta\mu = {} & \frac{3e^2}{64m_\pi F_\pi^2}\,\mathrm{Re}\,\mathrm{i}\int_{a-\mathrm{i}\infty}^{a+\mathrm{i}\infty}\frac{\mathrm{d}z}{\sin^2\pi z\tan\pi z}\quad\frac{g(z)+h(z)}{\Gamma(z)}\left(\frac{z-1}{z+1}\right) \\ & \times\left\{\int_0^\infty \mathrm{d}m^2\left(\frac{\kappa m}{4\pi}\right)^{2z}(\rho_{\mathrm{V}}-\rho_{\mathrm{A}}-\Delta\rho_{\mathrm{A}})-m_\pi^2F_\pi^2\left(\frac{\kappa m_\pi}{4\pi}\right)^{2z}\right\}. \end{aligned} \tag{15}$$

Since the existence of the moments of the spectral functions is probably a strong assumption, we first integrate over m^2.

Consider the sum rule

$$\int_0^\infty \mathrm{d}m^2(\rho_V(m^2)-\rho_A(m^2)-\Delta\rho_A(m^2)) = bm_\pi^2F_\pi^2. \tag{16}$$

For $b = 0$, $\Delta\rho_A = 0$ (16) is the second Weinberg sum rule, $b = 1$ gives the modification of Cook *et al.* If $b < \infty$ the spectral integral exists in (15) and the contour integration can be carried out. For $b = 0, 1$ and b near 1, (16) holds almost equally.

3. Evaluation of equation (15)

In the ρ_1, A_1 meson approximation

$$\rho_V = \left(\frac{m_\rho^2}{f_\rho}\right)^2\delta(m^2-m_\rho^2), \qquad \rho_A = \left(\frac{m_A^2}{f_A}\right)^2\delta(m^2-m_A^2), \qquad \Delta\rho_A = 0 \tag{17}$$

and (15) gives from the double pole at $z = 0$

$$\begin{aligned}\delta\mu = \frac{-3e^2}{32\pi^2m_\pi F_\pi^2}&\left\{\frac{m_\rho^4}{f_\rho^2}\ln\left(\frac{\kappa m_\rho}{4\pi}\right)^2-\frac{m_A^4}{f_A^2}\ln\left(\frac{\kappa m_A}{4\pi}\right)^2-m_\pi^2F_\pi^2\ln\left(\frac{\kappa m_\pi}{4\pi}\right)^2\right.\\ &\left.+\left(\frac{m_A^4}{f_A^2}-\frac{m_\rho^4}{f_\rho^2}+m_\pi^2F_\pi^2\right)(2+\psi(1)-g'(0)-h'(0))\right\},\end{aligned} \tag{18}$$

$$\psi(1) = \left.\frac{\mathrm{d}\ln\Gamma(z)}{\mathrm{d}z}\right|_{z=1}.$$

One can verify that the contributions coming from the points $z = 1, 2, \ldots$ are negligible, being of the order $\kappa^2, \kappa^2\ln\kappa^2, \kappa^2\ln^2\kappa^2$. To estimate $g'(0)+h'(0)$ we first note that $G(H)$ and $g(h)$ are connected by a Mellin transformation, for $c > 1$

$$\begin{pmatrix}g(z)\\ h(z)\end{pmatrix} = \frac{1}{\Gamma^2(z+1)\Gamma(-z)}\int_0^\infty \mathrm{d}u u^{-z-1}\begin{pmatrix}G(u)\\ H(u)\end{pmatrix}. \tag{19}$$

Taking into account (11) (Ashmore and Delbourgo 1971), we get a slow $\ln c$ type dependence from (19) to (18). For instance, for $c \lesssim 100$, $g'+h'$ contributes no more than 5% to (18), the gravitational gauge dependence of (18) is negligible.

In (18) f_A is replaced from the first Weinberg sum rule (14). To diminish the number of parameters we make use of the KSFR relation (Kawarabayashi and Suzuki 1966, Fayazuddin and Riazuddin 1966)

$$\frac{m_\rho^2}{2f_\rho^2} = F_\pi^2, \tag{20}$$

in such a way

$$\begin{aligned}\delta\mu = \frac{3\alpha}{8\pi m_\pi}\Bigg[&2m_\rho^2\ln\frac{m_A^2}{m_\rho^2}+(m_A^2-2m_\rho^2)\\ &\times\left\{\ln\left(\frac{\kappa m_A}{4\pi}\right)^2-\psi(1)-2\right\}+m_\pi^2\left\{\ln\left(\frac{\kappa m_\pi}{4\pi}\right)^2-\psi(1)-2\right\}\Bigg].\end{aligned} \tag{21}$$

The first term gives the old current algebra result (Das *et al* 1967), the others represent the regularizing effects. For $m_A^2 = 2m_\rho^2$ and $\kappa \to 0$ (21) reproduces exactly the logarithmic divergence coming from the pion pole discussed recently (Cook *et al* 1968), otherwise the logarithmic factors are about -90.

From (14), (16), (17) and (20) it follows that

$$m_A^2 - 2m_\rho^2 = -bm_\pi^2 \tag{22}$$

which holds well with $b \sim 0$–2. Now, combine (22) and (21); special cases: (i) $b = 0$, the large pion term survives, $\delta\mu \simeq -6$ MeV. (ii) A good solution arises from $b = 1$ (Cook *et al* 1968), $m_A = 1070$ MeV then $m_\rho = 763$ MeV, $\delta\mu = 4{\cdot}6$ MeV. In this case gravity is only an intermediate regulator, it is completely cancelled in the leading term (21). The same $\delta\mu$ follows without (20) from (14), (16), (17), $b = 1, m_A = 1070$ MeV, $F_\pi = 94$ MeV moving m_ρ between 760 MeV and 775 MeV. (iii) There exists a class of solutions where $b \neq 1$ and gravity contributes to the leading $\delta\mu$. For instance, when b is near one, (21) gives about 4·6 MeV while the limit $\kappa \to 0$ diverges. Equation (21) gives in itself $\delta\mu = 8{\cdot}2$ MeV for $m_\rho = 765$ MeV, $m_A = 1070$ MeV.

4. Discussion

In the present paper we have computed the electromagnetic mass difference of pions to second order in the electromagnetic interaction, taking into account the strong interaction in a lagrangian-independent way, and in the presence of tensor gravity. The localizable exponential parametrization approximates the nonlocalizable parametrization mentioned after equation (12), since in both cases the mass shifts are the same as is seen from equation (18). The violation of the electromagnetic and gravitational gauge invariances is weak.

The introduction of gravity modifies the photon propagator and makes $\delta\mu$ finite, even in the cases of 'wrong' sum rules where for $\kappa \to 0$ $\delta\mu$ diverges. It turns out that $\delta\mu$ is sensitive to $m_A^2 - 2m_\rho^2$ and to the sum rules predicting this quantity. At present, the experimental value of $\delta\mu$, 4·6 MeV, does not choose a sum rule as the only possible one. Good solutions for $\delta\mu$ arise from complete or partial cancellations of the gravitational terms. In this connection let us note the superconvergence assumption leading to (16). Write (6) as

$$\Delta^V_{\nu\sigma}(p) - \Delta^A_{\nu\sigma}(p) = \eta_{\nu\sigma}F(p^2) - p_\nu p_\sigma G(p^2). \tag{23}$$

The second Weinberg sum rule comes from $p^2F(p^2) \to 0$ at $p^2 \to \infty$ while at $b = 1$ (16) is given by the superconvergence assumption $p^4G(p^2) \to 0$ at $p^2 \to \infty$ (Cook *et al* 1968). On the same basis one may require

$$xp^2F(p^2) + yp^4G(p^2) \to 0, \qquad p^2 \to \infty \tag{24}$$

which leads to (16)

$$\int_0^\infty dm^2(\rho_V(m^2) - \rho_A(m^2) - \Delta\rho_A(m^2)) = \left(\frac{y}{x+y} - \frac{x}{x+y}\frac{z}{m_\pi^2 F_\pi^2}\right) m_\pi^2 F_\pi^2,$$

$$z = \int_0^\infty \Delta\rho_A(m^2)\, dm^2. \tag{25}$$

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